

Title of dissertation: Cantor's and Dedekind's philosophies of mathematics. A comparative analysis

Abstract:

This dissertation is a comparative case study of two individual mathematical practices and the selected philosophical aspects of the work of the Georg Cantor and Richard Dedekind. This study is part of the current of research on the development of mathematical knowledge, where the need for extending knowledge with case studies, which are focused on specific mathematical problems solved by specific mathematicians, is noted. The dissertation aims to meet this need, based on the analysis of historical examples. To this end, a comparative analysis of the philosophical assumptions of analysed mathematicians of the 19th century, is made. This research direction is inspired by Michael Heller's postulate on the need to broaden the range of questions and research issues of the philosophy of mathematics. Heller's programme is developed with reference to the philosophy of mathematical practice (PMP).

The main aim of the paper is to attempt to challenge the radical prevailing view of Cantor-platonic vs Dedekind-constructivist. Within the research stream of mathematical practice, this paper argues for a pragmatic approach that makes the emergence of mathematical objects dependent on the concrete, mental constructional activities of the subject in response to specific mathematical problems in an individual and social perspective. This perspective allows that, from the textual and linguistic introduction of these objects, they become in some sense intersubjectively communicable.

The paper focuses on the following mathematical contents: 1. on the problem of the construction of the discrete continuum in both mathematicians, 2. in Cantor's case on the emergence of infinite sets, infinite numbers - cardinal and ordinal, as well as the proof against infinitesimals, 3. and in Dedekind's case on an outline of the foundations of set theory, as well as on the arithmetic of natural numbers. Selected mathematical problems are analysed in the context of contemporary proposals.

In addition to the analysis of mathematical texts, the reconstruction and comparative analysis of philosophical assumptions and premises, an attempt has been made to characterise selected aspects of the methodology of the scientific practice of the two mathematicians, based on their scientific decisions and research directions taken, in the context of the scientific environment of the time and the paradigms in force in that environment. To this end, their assumptions about the source of mathematical knowledge, their approach to the use and

application of set theory are analysed, and selected aspects of their mathematical view of the foundations of mathematics are described.

The analyses undertaken allow us to reconstruct three important aspects of Cantor's and Dedekind's philosophy of mathematics. These are epistemological constructionism, ontological realism, and related structuralism - in Cantor's case only ontological structuralism, in Dedekind's additionally methodological-epistemological structuralism. The proposed epistemological constructionism is based on the indispensability of the active and creative mental processes of the mathematical subject in the context of constructing mathematical structures; realism is based on the assumption of the indispensability of the minimal requirement of abstract objects that are the product and object of discourse of both individual practice and socio-historical practices.

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